14231 120 MINUTES

1. For the set $E = \left\{\frac{1}{n} : n \in N\right\}$, which of the following statements is not true?

A) The element 1 is an upper bound of E
B) The set E has a minimum
C) All negative numbers and 0 are lower bounds of E
D) No element of E can be a lower bound of E

2. $\lim_{n \to \infty} \left[\frac{1}{\sqrt{n+1}} + \frac{1}{\sqrt{n+2}} + \cdots + \frac{1}{\sqrt{n+n}}\right]$ equals

A) ∞ B) $\frac{1}{\sqrt{2}}$ C) $\sqrt{2}$ D) $\frac{1}{2\sqrt{2}}$

3. The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges for

A) $p< 1$ B) $p = 1$ C) $p > 1$ D) $p \leq 1$

4. Let $f(x) = \begin{cases} 2x+1, \text{ for } x \leq 1 \\ 2x^2 + a, \text{ for } 1 < x < 3 \\ 5x+4, \text{ for } x \geq 3 \end{cases}$

be continuous everywhere. Then the value of a is

A) 0 B) 1 C) 2 D) 3

5. Which of the following in not true?

A) Every open interval is open
B) The set R of all real numbers is open
C) The union of any collection of open sets is open
D) A subset E of R is open if its complement is open

6. If C is $|z| = 2$, then value of the integral, $\int_C \frac{1}{2z+3} dz$ is

A) $-2\pi i$ B) $-\pi i$ C) $2\pi i$ D) πi

7. Order of the pole at $z = 0$ of the function $\frac{1 - e^{2z}}{z^4}$ is

A) 4 B) 3 C) 2 D) 1

8. The residue of $\frac{e^{2z}}{(z-1)^2}$ at $z = 1$ is

A) $2e^2$ B) e^2 C) $\frac{e^2}{2}$ D) $3e^2$

9. Let the characteristic equation of a nonsingular matrix A be $\lambda^2 - 4\lambda + 4 = 0$. Then A^{-1} equals

A) $I - 4A$ B) $I + \frac{1}{4}A$ C) $I - \frac{1}{4}A$ D) $I + 4A$

10. If 2, 5 and 8 are the eigen values of a matrix A of order 3, then the value of |A| equals

A) 15 B) –15 C) 80 D) –80

11. If A is a matrix of order 3 and |2A| = k|A|, then the value of k equals

A) 8 B) 4 C) 16 D) 2

12. IfA and B are two symmetric matrices of the same order,then which of the following is not necessarily a symmetric matrix?

A) A + B B) A – B C) AB D) $A + B^T$

13. IfA $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$, then the characteristic equation is given by

A) $\lambda^3 + \lambda^2 + \lambda + 1 = 0$ B) $\lambda^3 + 3\lambda^2 + 3\lambda + 1 = 0$

C) $\lambda^3 - \lambda^2 - \lambda + 1 = 0$ D) $\lambda^3 - 3\lambda^2 + 3\lambda + 1 = 0$

14. The algebraic multiplicity of the eigen value 2 of the matrix $\begin{bmatrix} 3 & -1 & 1 \\ 7 & -5 & 1 \\ 6 & -6 & 2 \end{bmatrix}$ is given to be 2. Which of the following numbers can be the geometric multiplicity of 2?

A) 5 B) 4

C) 3 D) 1

15. Let V be a vector space of dimension 12 and S, a subspace of dimension 4. What is the dimension of V/S?

A) 6 B) 3

C) 8 D) 4

16. $\{A_n\}$ is a sequence of sets such that $A_n = \begin{cases} A, \text{if } n \text{ is odd} \\ B, \text{if } n \text{ is even} \end{cases}$

Where A and B are two non empty sets. Now consider the following

1. $\underline{\lim} A_n = A \cup B$
2. $\lim A_n = A \cap B$

Which of the above statements is/are true?

A) 1 only B) 2 only

C) 3 only D) None of these

17. For any sequence $\{A_n\}$ of sets, consider the following

1. $\underline{\lim} A_n \subset \overline{\lim} A_n$
2. $\left(\overline{\lim} A_n\right)^c = \underline{\lim} A_n^c$

Which of the above statements is/are true?

A) 1 only B) 2 only
C) Both 1 and 2 D) None of these

18. A class of subsets of a non-empty set is a σ - field if it is closed under
A) Complementation only
B) Countable union only
C) Complementation and countable union
D) Complementation and finite union

19. Let $\{f_n\}$ be a sequence of measurable functions which is bounded below by an integrable function. Then which of the following holds good?

A) $\int \liminf f_n d\mu \leq \lim \sup \int f_n d$
n→∞n→∞

B) $\int \lim \sup f_n d\mu \geq \liminf \int f_n d$
n→∞n→∞

C) $\int \liminf f_n d \leq \liminf \int f_n d$
n→∞n→∞

D) $\int \lim \sup f_n d\mu \geq \lim \sup \int f_n d$
n→∞n→∞

20. Let * be an outer measure on Ω and let M bethe class of subsets of Ω which are measurable w.r.t. *. Consider the following statements
1. M is a σ field. 2. Restriction of μ* to M does not define a measure

Which of the above statements is/are true?

A) 1 only B) 2 only
C) Both 1 and 2 D) None of these

21. A real valued set function P defined on a σ-field of subsets of the sample spaceΩ of a random experiment is a probability measure if
A) P is nonnegative
B) P(Ω) = 1
C) P is countablyadditive
D) P satisfies conditions A),B)& C)

22. Let (Ω, $\mathcal{A}$, P) be a probability space. Consider the following statements

1. P(A) = 0 for some A $\in \mathcal{A}$ implies that A = ϕ, the null event
2. P(B) = 1 for some B $\in \mathcal{A}$ implies that A = Ω

Which of the above statements is/are true?

A) 1 only B) 2 only
C) Both 1 and 2 D) None of these

23. Let (Ω, $\mathcal{A}$, P) be a probability space and $\{A_n\}$, a nondecreasing sequence of events. Which of the following is true?

A) $\lim_{n\to\infty} P(A_n) = P\left(\bigcap_{n=1}^{\infty} A_n\right)$

B) $\lim_{n\to\infty} P(A_n^c) = P\left(\bigcup_{n=1}^{\infty} A_n^c\right)$

C) $\lim_{n\to\infty} P(A_n) = P\left(\bigcup_{n=1}^{\infty} A_n\right)$

D) $\lim_{n\to\infty} P(A_n^c) = P\left(\bigcap_{n=1}^{\infty} A_n\right)$

24. Consider families with two children and assume that all possible distributions of gender are equally likely. Let E be the event that a randomly chosen family has atmost one boy and F, the event that the family has both genders. Then which of the following is true?

A) E and F are independent
B) E and F are not independent
C) E and F are mutually exclusive
D) E and F are independent and mutually exclusive

25. For any two events A and B defined on a probability space, $P(A \cap B) \geq P(A) + P(B) - 1$. This result is known as

A) Bonferroni's inequality B) Boole's inequality
C) Subadditive of P D) Monotone property of P

26. Let A and B be two independent events defined on some probability space and let P(A) = 1/3, P(B) = ¾. Then the value of P(A U B) is

A) $\frac{5}{6}$ B) $\frac{7}{12}$ C) $\frac{3}{12}$ D) $\frac{2}{3}$

27. Ten people are randomly seated at a round table. What is the probability that a particular couple will sit next to each other?

A) $\frac{1}{10}$ B) $\frac{1}{9}$ C) $\frac{2}{10}$ D) $\frac{2}{9}$

28. Let $\{A_n\}$ be a sequence of events defined on a probability space $(\Omega, \mathcal{A}, P)$ such that $\sum_{n=1}^{\infty} P(A_n) = \infty$. Then the value of $P(\limsup A_n)$ is

A) 1
B) 0
C) A real number between 0 and 1
D) 1 only when the sequence of events is independent

29. Kolmogorov inequality for a random variable with mean 0 and variance σ^2 reduces to

A) Chebychev's inequality
B) Markov inequality
C) Lyapunov inequality
D) Jenson's inequality

30. Consider the following results on strong law of large numbers (SLLN)

1. Every sequence of random variables with uniformly bounded variance obeys SLLN
2. Every sequence of i.i.d. bounded random variables obeys SLLN

Which of the above results is/are true?

A) 1 only
B) 2 only
C) Both 1 and 2
D) None of these

31. Let X_1, X_2, --- be a sequence i.i.d random variables having the Bernoulli's distribution with parameter p and $S_n = \sum_{j=1}^{n} X_j$, for $n \geq 1$. Which of the following sequences of random variables converges to the standard normal distribution?

A) $\frac{S_n - np}{\sqrt{pq}}$
B) $\frac{S_n - np}{\sqrt{npq}}$
C) $\frac{S_n - p}{\sqrt{pq}}$
D) $\frac{S_n - p}{\sqrt{npq}}$

32. Let X_1, X_2, --- be uniformly bounded independent random variables with $V(X_j) = \sigma_j^2$, $j = 1, 2, ---$ and let $s_n^2 = \sum_{j=1}^{n} \sigma_j^2, n \geq 1$. A necessary and sufficient condition for the Lindberg Feller central limit theorem to hold is

A) $s_n^2 \to \infty$ as $n \to \infty$
B) $(1/s_n^2) \to \infty$ as $n \to \infty$
C) $s_n^2 \to 0$ as $n \to \infty$
D) $s_n \to 0$ as $n \to \infty$

33. For the Bernoulli distribution, the value of $\beta_2 - \beta_1 - 1$ is

A) 1
B) 0
C) 1/2
D) -1/2

34. For which of the following distributions, mean is less than variance

A) Binomial
B) Geometric
C) Poisson
D) The standard exponential

35. Suppose X has a Poisson distribution with third central moment, 1. Then the mean and standard deviation of the distribution are respectively

A) 1 & 1 B) 1 & 1/2
C) 1/2 & 1 D) 1/2 & 1/2

36. Let X follow binomial (n,p) andY, the negative binomial (r,p). Which of the following relations is true?

A) $P(X \leq r-1) = P(Y \leq n-r)$ B) $P(X \leq r-1) = P(Y > n-r)$
C) $P(X \leq n-r) = P(Y \leq r-1)$ D) $P(X \leq n-r) = P(Y > r-1)$

37. Let X ~ U(0,1) and given that the conditional distribution of Y given X = x is binomial (n,x). Then the expected value of Y is

A) n/2 B) n
C) n/3 D) 2n

38. If X ~ U (0,1), what is the distribution of $-2\ ln\ X$?

A) Lognormal B) Exponential with mean 2
C) Double exponential D) Exponential with mean 1

39. If X follows exponential distribution with mean 2, what is the distribution of $1 - e^{-\frac{x}{2}}$?

A) Standard normal B) Exponential with mean 2
C) Uniform over (0,1) D) Standard Cauchy

40. What is the fourth central moment of normal distribution with variance 4?

A) 16 B) 48
C) 12 D) 18

41. Let X_1, X_2 be i.i.d. standard normal variates. Then the distribution of $(X_2 - X_1)^2/2$ is

A) Chisquare with 1 d.f B) Chisquare with 2 d.f
C) Standard normal D) Standard Cauchy

42. Let $X_1 \sim N(0,1)$ and $X_2 \sim N(0,2)$. Then for positive $\in$, which of the following is correct?

A) $P(|X_1| < \in) < P(|X_2| < \in)$ B) $P(|X_1| < \in) > P(|X_2| < \in)$
C) $P(|X_1| < \in) = P(|X_2| < \in)$ D) $P(|X_1| < \in) \leq P(|X_2| < \in)$

43. For $X \sim N(\mu,\sigma^2)$, mean deviation of X is

A) Equal to σ B) Larger than σ
C) Smaller than σ D) Not necessarily finite

44. If $X_{(1)}$, $X_{(2)}$, $X_{(3)}$ are the order statistics of three independent random variables drawn from the exponential distribution with mean 1/λ, then the distribution of $Y_1 = X_{(3)} - X_{(2)}$ is

A) Exponential with mean 3λ B) Exponential with mean 3/λ
C) Exponential with mean λ D) Exponential with mean 1/λ

45. If $X_{(1)}$, $X_{(2)}$, $X_{(3)}$, $X_{(4)}$, $X_{(5)}$ are the order statistics of five i.i.d. U(0,1)-variates and $\beta_j(m,n)$ denotes beta distribution of the jth kind with parameters m and n, for j = 1, 2, then the distribution of $X_{(3)}$ is

A) β_1 (3,3) B) β_2 (3,3) C) β_1 (2,3) D) β_2 (2,3)

46. If $X_{1:5}$, $X_{2:5}$, $X_{3:5}$, $X_{4:5}$, $X_{5:5}$ are the order statistics of a random sample of size 5 drawn from a population with an absolutely continuous distribution function F(x), then the conditional distribution of $X_{4:5}$ given $X_{2:5}$ = x is the same as the distribution of the order statistic

A) $X_{1:3}$ arising from a population with distribution F(x) truncated on the left at x

B) $X_{1:3}$ arising from a population with distribution F(x) truncated on the right at x

C) $X_{2:3}$ arising from a population with distribution F(x) truncated on the left at x

D) $X_{2:3}$ arising from a population with distribution F(x) truncated on the right at x

47. If t ~ Student's $t_{(n)}$, what is the distribution of t^2?

A) F(1,n) B) F(n,1) C) F(1,1) D) F(n,n)

48. If X_1, X_2, X_3 are independent standard normal variates, then the distribution of

$$Y = \frac{\sqrt{2}\,X_1}{\sqrt{X_2^2 + X_3^2}}$$ is

A) Student's $t_{(1)}$ B) Standard normal

C) Student's $t_{(2)}$ D) F(1, 3)

49. Let X be a single observation taken from a population having Poisson distribution with parameter θ. Then an unbiased estimator of (1 + θ) (2 +θ) is

A) $3X^2 + 9$ B) (1 + X) (2 + X)

C) $X^2 + X + 2$ D) $X^2 + 2X + 2$

50. The estimates t_1 and t_2 are given to be unbiased for the parameters θ_1 and θ_2 respectively. If t_1 and t_2 are independently distributed, which one of the following statements is not correct?

A) $t_1 + t_2$ is unbiased for $\theta_1 + \theta_2$ B) $t_1 - t_2$ is unbiased for $\theta_1 - \theta_2$

C) t_1 x t_2 is unbiased for θ_1 x θ_2 D) $\frac{t_1}{t_2}$ is unbiased for $\frac{\theta_1}{\theta_2}$

51. Consider a population with the pdf, $f(x,\theta) = \begin{cases} \theta e^{-\theta x}, & 0 \le x < \infty; \theta > 0 \\ 0, & \text{otherwise} \end{cases}$. What is the m.l.e. of θ based on a sample of size n drawn from the above population?

A) $\overline{X}$ B) $\frac{1}{\overline{X}}$ C) $X_{(1)}$ D) $X_{(n)}$

52. Let t_n be an unbiased and consistent estimator of θ. Then, as an estimator of θ^2, t_n^2 is
A) Unbiased but not consistent B) Biased but consistent
C) Biased and not consistent D) Unbiased and consistent

53. Let X and Y be independent random variables with the same mean θ and same variance 36. For θ, two unbiased estimators T_1 and T_2 are given by $T_1 = \frac{2X+Y}{3}$ and $T_2 = \frac{X+Y}{2}$. Then the relative efficiency of T_1 w.r.t. T_2 is
A) 1 B) 9/10
C) 2/5 D) 3/5

54. If a statistic $t = t(x_1, ---, x_n)$ provides as much information about the population parameter θ as the random sampte $(x_1, ---, x_n)$ does, then t is called
A) An unbiased estimator B) A consistent estimator
C) An efficient estimator D) A sufficient estimator

55. Let U be an unbiased estimator θ and S, a complete sufficient statistic. If T = E(U|S), which of the following is/are true?
A) T is an unbiased estimator for θ
B) T is the unique UMVUE of θ
C) T is a function of S, which is independent of θ
D) All of these

56. If $X_1, X_2, ---, X_n$ is a random sample from the Poisson population with mean λ, the Cramer Rao lower bound to the variance of any unbiased estimator of λ is given by
A) $\frac{\sqrt{\lambda}}{n}$ B) $\frac{\lambda^2}{n}$ C) $\frac{\lambda}{n}$ D) $\frac{e^{-\lambda}}{n}$

57. The following is a random sample taken from a population with uniform distribution over (0,θ): 0.1, 3.7, 5, 4.1, 4.5. Then the maximum likelihood estimate of θ^2 is
A) 5 B) 0.1 C) 25 D) 4.5

58. The moment estimator of θ based on a random sample of size n from a population with uniform distribution on (0,θ), θ > 0 is
A) 2x sample mean B) Sample mean
C) Sample median D) Sample maximum

59. The observations 10, 20, 24, 21, 16, 14, 27, 12, 18 are recorded at random from a normal population with mean and standard deviation 9. What is the shortest 95% CI for ?
A) (13.05, 22.95) B) (12.12, 23.88)
C) (15, 21) D) (9, 27)

60. Let $X_1, X_2, ---, X_n$ be i.i.d.N (μ, σ^2), where both μ, σ^2 are unknown. Which of the following is not a composite hypothesis?

A) = 0 B) $\sigma^2 = 4$
C) $\mu \geq 0, \sigma^2 = 4$ D) $\mu = 0, \sigma^2 = 4$

61. Consider testing of $H_0 : \theta = 2$ against $H_1 : \theta = 1$ based on a single observation X_1 from the population $f(x;\theta) = \theta e^{-\theta x}$, $x \geq 0$; $\theta > 0$. Let $X_1 \geq 1$ be the critical region. Then power of the test is

A) $\frac{1}{e}$ B) $\frac{1}{e^2}$ C) $1 - \frac{1}{e^2}$ D) $\frac{e-1}{e}$

62. Let $X_1, X_2, ---, X_n$ be a sample from $N(0, \sigma^2)$ and let $H_0 : \sigma = \sigma_0$. Consider the following statements for testing H_0:
1. If $H_1 : \sigma > \sigma_0$, a UMP test exists
2. If $H_1 : \sigma \neq \sigma_0$, no UMP test exists.

Which of these statements is/are true?

A) 1 only B) 2 only
C) Both 1 and 2 D) None of these

63. To test the hypothesis that a sample of observations arises from a specified distribution against the alternative that it is from some other distribution, which of the following tests is used?

A) Sign test B) Wilcoxon-signed ranks test
C) Kolmogorov-Smirnov test D) Mann-Whitney test

64. For the SPRT of strength (α, β), which of the following inequalities is satisfied by the stopping bounds A and B (A > B)?

A) $A \geq \frac{1-\beta}{\alpha}, B \leq \frac{\beta}{1-\alpha}$ B) $A \leq \frac{1-\beta}{\alpha}, B \geq \frac{\beta}{1-\alpha}$
C) $A \geq \frac{1-\alpha}{\beta}, B \leq \frac{\alpha}{1-\beta}$ D) $A \leq \frac{1-\alpha}{\beta}, B \geq \frac{\alpha}{1-\beta}$

65. The usual one way analysis of variance under a fixed effects model is a test for the comparison of several

A) Variances B) Means
C) Correlation coefficient D) Medians

66. Consider the following analysis of variance table

Source	df	SS
Factor A	2	20
Factor B	3	30
Interaction	-	36
Error	10	40

What is the calculated value of the F-ratio for interaction effect?

A) 1 B) 1.5 C) 2 D) 2.5

67. For a BIBD with parameters $\upsilon = b = 4$, $r = k = 3$, the number of treatments common between any two blocks is

A) 4 B) 3 C) 2 D) 1

68. Analysis of variance in the form of RBD is carried out when it is known that within blocks and between blocks there is
A) Homogeneity and heterogeneity respectively
B) Heterogeneity and homogeneity respectively
C) Homogeneity in both
D) Heterogeneity in both

69. Consider the following linear forms in y1, y2, y3:
I. $y_1 - 2y_2 + 2y_3$ II. $y_1 - 2y_2 + y_3$ III. $3y_1 + 4y_2 - 7y_3$
Which of these forms is/are linear contrasts?

A) Only I B) II and III
C) I and III D) Only III

70. For a 2^3-factorial design with 2 replications, what is the error df?
A) 1 B) 6
C) 7 D) 8

71. In a RBD with r blocks, t treatments and one missing observation, the d.f. for the error SS is
A) rt – r – t B) rt – r – 1
C) rt – r – t + 1 D) rt – r

72. In a symmetric BIBD with usual notations if υ is even, then $r - \lambda$ is
A) An even number B) An odd number
C) Zero D) A perfect square

73. If a sample of size 2 is drawn from a population of size 3 by the method of SRSWR, then the probability of selecting a specified unit in any sample would be
A) $\frac{1}{9}$ B) $\frac{4}{9}$ C) $\frac{5}{9}$ D) $\frac{2}{3}$

74. A simple random sample of 5 households was drawn from a village containing 125 households. The number of persons per household in the sample were4, 5, 6, 7 and 3. The estimate of the total number of people in the village is
A) 25 B) 625 C) 1250 D) 3125

75. Let a population of size 5 have its mean $\overline{Y} = 12$ and $S^2 = 100$. A sample of size 2 is drawn without replacement. If the sample mean is $\overline{y}$ then $E(\overline{y}^2)$ is
A) 30 B) 50 C) 144 D) 174

76. If a stratified sample of 45 units is to be selected by Neyman allocation from a population with $N_1 = 150$, $N_2 = 350$, $S_1^2 = 4$, $S_2^2 = 9$, then the number of units to be selected from the first stratum is

A) 10 B) 20 C) 35 D) 75

77. From a population of 23 units, a sample of 4 units is to be selected by systematic sampling. If 8th unit is selected, then what are the other units?

A) 13,19,3 B) 12,18,2 C) 14,20,3 D) 13,17,22

78. It is given that N = 1000, n = 100, $\bar{x} = 250$, $\bar{y} = 500$, $\bar{X} = 275$. Then the ratio estimator of $\bar{Y}$ is

A) 525 B) 550 C) 575 D) 625

79. A finite population is divided into three strata. The sizes of the first, second and third strata are 20, 40, x respectively. A stratified random sample is drawn from the population using proportional allocation. If the total sample is 30 and the sample size for the first stratum is 6, then x equals

A) 80 B) 60 C) 40 D) 20

80. In population with linear trend with N =nk, consider the following variances of the sample mean:

1. V_{sy} (in the case of systematic sampling)
2. V_{st} (in the case of stratified sampling)
3. V_{ran} (in the case of simple random sampling)

The correct arrangement of the above variances in the increasing order is

A) 1, 2, 3 B) 1, 3, 2 C) 2, 1, 3 D) 3, 1, 2

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